

Classification of Sequential Sports Using Automata Theory

Sravya Gurram*

M. Sc. Student, Human Sciences Research Group of Dr. Aniket Alam,
International Institute of Information Technology-Hyderabad, Hyderabad, India

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Abstract

This paper proposes a categorization of sport that is based on the system of rules that a sport must adhere to. We focus on these systems of rules to examine how a winner is produced in different sports. The rules of a sport dictate the gameplay and the direction it takes. We propose to break down the gameplay into Events. At this junction, we observe two kinds of events – ones that follow sequential logic and ones that do not. Our focus is pertained to sports that comprise sequential events. To examine these events further, to understand how a winner emerges, we take the help of Finite State Automaton from theory of computation (Automata theory). We showcase how sequential sports are eligible to be represented as Finite State Machines. We depict these Finite State Machines as State Diagrams. We examine these State Diagrams to observe how a team/player reaches the final states of the sport, with special focus on one final state – the final state which determines the winner. This exercise has been carried out for the following sports: Hurdles, Track, Shot Put, Long Jump, Bowling, Badminton, Pacman, Weightlifting (Snatch). Based on our observations of how this final State of winning is achieved, we propose a categorization of sports.

Keywords: Sport Classification; Sport Modelling; Ontology; Knowledge Representation

* **Author's e-mail:** sravya.gurram@research.iiit.ac.in (**Corresponding Author**)

INTRODUCTION

In sports, Individuals or teams enter a secondary reality with time and space constraints and emerge as a winner and losers. The space, time constraints are the rules that the participants of a sport must adhere to. These rules are different in different sports to varying degrees, but they have in common that they are designed so that a winner emerges from within the participants. In the course of this paper, we analyze the various ways in which different sports are designed so a winner is chosen. We undertook this exercise to propose a classification of sports that has the system of rules of sports as the classifying parameter. For the purpose of this study, we have chosen the following Sports: Hurdles, Track, Long Jump, and Shot put, Badminton, Bowling, Pacman and Weightlifting.

Literature Review

In *Man, Play and Games*, a seminal work in the sociology of Play and Games, Roger Caillois proposes a list of essential features for an activity to be deemed as a game as well as proposes a classification of games. The definitive qualities of a Game presented in this book are heavily influenced by Johann Huizinga's *Homo Ludens: A Study of Play Element in Culture*. It is through the criticism of Huizinga's definitive properties of Play, i.e., 'free, separate, uncertain, unproductive, regulated and fictive' (Gillin & Huizinga, 1951) that Caillois arrives at his essential features of a Game –

- *Free*: Caillois describes play as a free activity in which no part is obligatory.
- *Separate*: Believing in rules creates fiction. Caillois points out that by adhering to the rules of a game, we enter a suspended reality that is separate from real life.
- *Uncertain*: The course of the gameplay must remain uncertain, i.e., the outcome cannot be predetermined.
- *Unproductive*: Gameplay must not create property of any kind, but it may include exchange of already existing properties as a part of the gameplay. (Example: Gambling games)
- *Governed by rules*: While playing, the players must suspend the rules of real life and adhere to the rules of the game.

- *Make-believe*: A Game is always accompanied by an awareness of a secondary reality, separate from real life.

Caillois proposes a classification of games that is widely accepted because it can fit a wide variety of games under its purview. The categories of games are as follows: *Agon*, *Alea*, *Mimicry* and *Ilinx*. (Caillois, 2001) For games that fall under *Agon*, competition between teams or individuals to establish a superior is the central theme. Games of the *Alea* kind depend on the throw of the dice, or other elements not under the control of the participants of the game. *Mimicry Games* constitute those games that involve imitation while *Ilinx* games are those that seek for a distortion of reality by rapid movements.

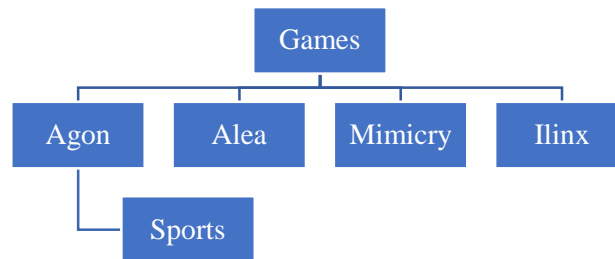


Figure 1: Classification of Games proposed by Roger Caillois in *Man, Play and Games*

Sports, the kind of games that are in focus in this paper fall under the category of *Agon*. According to Caillois, *Agon* games (Sports) artificially portray an equality of chances so that the opposing teams or players confront each other until a winner emerges. The equality of chances between the participants for one of the participants to emerge as a winner is enforced by the system of rules that every Sport must adhere to.

In this paper, we focus on how *Agon* games, in particular Sports, arrive at the concluding final state - i.e., the winner. Based on these observations, we propose a classification of sports that is also an extension of the classification of games proposed by Roger Caillois. The Sports that we have taken into consideration for the purpose of the above-mentioned exercise are Hurdles, Track, Shot Put, Long Jump, Bowling, Badminton, Pacman, Weightlifting (Snatch). In the next section of this paper, we outline the

methodology we assumed to analyze how the above-mentioned sports arrive at their concluding states.

Methodology for the Analysis of Sports

For the analysis of sports, we begin with a rather simplistic view of sport, i.e., all participants must adhere to a system of rules and reach a concluding state where a winner is announced.

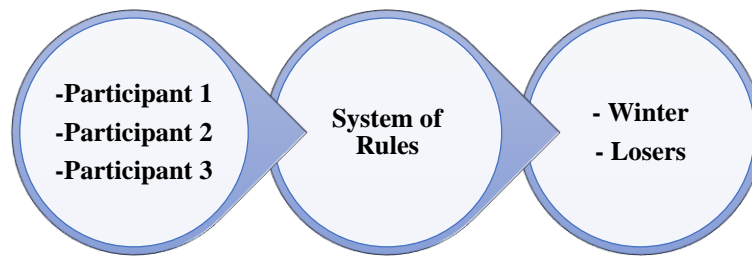


Figure 2: Abstract Representation of Sports

From the above representation of sport, it is obvious that to analyze how a winner is produced, we must begin by analyzing the system of rules that govern a sport. For this purpose, we have listed out the rules that govern the sports we have taken into consideration.

System of Rules

1. Hurdles

- Players must not start running at a false start.
- Player must jump over the hurdles
- Player must keep in his track
- Player must not knock the hurdles
- Player's leg must not go around the hurdle
- Player's leg must be above the horizontal plane of the top of the hurdle

2. Track

- Players must not start running at false start
- Players must not obstruct other players
- Players must stay within their track

3. Shot Put

- After taking position, the athletes must momentarily pause before throwing the shot
- The pause must not be longer than 1 minute.
- Athletes must not allow the shot to drop lower than his shoulder
- Athletes must not leave the circle until the shot has landed.
- The shot must land within the confines of the sector.

4. Long Jump

- Player must jump before crossing the foul line
- Player must land on both his feet

5. Bowling

- Player must not release the ball without crossing the foul line

6. Badminton

- Player must serve from within the service boundaries which are divided by the center line. Even number services take the right side of the court, while the odd numbered services take the left side of the court.
- Player must always serve diagonally to the opponent's court.
- The shuttle must cross the service line during service.
- The Players must return the rally without letting the shuttle touch the ground.
- Player must not touch the net with his body or racket at any point during the gameplay.
- The Players must contact the shuttle only with his racket.
- Player can touch the shuttle with his racket only once before returning the rally.
- Player can touch the shuttle with his racket only after the shuttle crosses the net.

7. Pacman

- Pac Man must eat all the Pac dots in the maze to move to the next level.
- Pacman should avoid getting killed by the monster

8. Weightlifting (Snatch)

- Lifter must not pause during the lifting of the barbell.
- Lifter must not stop the upward motion while the pull
- The barbell must not contact the lifter's head or shoulders while the upward motion of the pull
- Lifter must lift the barbell within 60 seconds of gripping the barbell
- Lifter must remain motionless while holding the barbell up
- Lifter must not bend or extend elbow while trying to remain motionless/recover with the barbell up
- Lifter must not let go of the barbell from above his shoulders
- Lifter must not lower the barbell before the referee signals
- Lifter must replace the complete barbell on the platform

From these rules, we can observe that within a sport, different rules are applied at different instances of the game play. For instance, the rule - 'Player's leg must not go around the hurdle' from the sport Hurdles, is a rule that is only applicable to the event of jumping over the hurdle during the game play. The rule cannot be applied to other events like Player running, reaching the end line etc. Based on this observation, we make a distinction to the system of rules that are applicable to a sport - Game rules and Event rules. Game rules are those that are applicable through the entirety or through most part of the game whereas Event rules are specific to certain events that constitute the gameplay of the sport.

To properly analyze these rules and how they facilitate the announcement of a winner, we need to obtain a much more detailed view of sports than just listing out its rules. We need to also list out the events that constitute the gameplay of these sports. We are undertaking this exercise to analyze the rules which apply to certain events or sequence of events so that it leads to its concluding state.

Events of Sports

1. Hurdles

- Player takes position at start line
- Player is running
- Player jumps over the hurdles
- Player reaches finish line

2. Track

- Player takes position at start line
- Player is running.
- Player reaches finish line.

3. Shot Put

- Player takes position in the circle
- Player throws the Shot
- Shot lands

4. Long Jump

- Player takes position at the start line
- Player is running
- Player jumps
- Player lands

5. Bowling

- Player takes position at the foul line
- Player released the ball
- The ball hits the pins
- The ball does not hit the pins

6. Badminton

- Player serves
- Shuttle crosses the net
- Shuttle does not cross the net
- Player strikes the shuttle with the racket
- Player lets the shuttle drop

7. Pacman

- Pacman start moving

- Pacman eats Pac dots
- Pacman gets killed by the monster

8. Weightlifting

- lifter grips the barbell & bends at the knee
- lifter pulls the barbell in a single movement to the full extent of both arms
- lifter becomes motionless while holding the position
- lifter lowers after referee's signal

FORMAL REPRESENTATION OF SPORTS

The above representation of sports which encapsulates the domain in the form of Events and Rules is a step ahead of the abstract representation of sports given in Figure 2 but shies away from being a formal representation of Sports. In this section, we aim to represent the domain of sports formally with the help of Automata theory (Computation Theory).

Finite State Automaton

A Finite State Automaton (or a Finite State Machine) is a computational representation of sequential logic. (*Finite State Machines | Brilliant Math & Science Wiki*, n.d.) A finite State Machine or Automaton captures the following:

- All the possible states that a system could possibly exist in
- All the possible of state changes that a system could carry out
- The inputs required for a system to change its state from one to another.

The automaton accepts a sequence of events, starting with the opening event of the sport and ends at any one of the termination points of the sport. At every event in the accepted sequence of events, the Finite State Automaton changes its state. It does so for every event of the accepted sequence and finally reaches one of its accepting (final) states. The FSA decides the output (termination point) based on the input of events given to the Automaton.

We can traverse the finite state machine diagram of a sport to see what sequence of events the sport will produce. Another advantage of using a finite state machine to represent sequential sports is that we can give as input

a sequence of events and verify whether or not there exists a set of transitions in that sport so that it ends in one of the accepting states. Each event is represented by a symbol, such that the sequence of events accepted by a Finite State Automaton are represented as a string of symbols.

Formally a Finite State Machine can be defined as a quintuple $T=(Q,\Sigma,\delta,q,F)$ where :

- Q is the finite and non-empty set of all states
- Σ is the finite set of all the symbols. Σ is called the 'Alphabet'.
- δ is the transition function which determines the next state given the input.
- q is the start state. State q is a part of the Q set.
- F is a set of all accepting states or final states. F is always a subset of Q .

Regular Language

A string of symbols accepted by a Finite State Automaton/Machine is formally referred to by the term Regular expression or 'regex'. The set of all regular expressions accepted by a Finite State Automaton is called the Regular Language of that Finite State Automaton. Regular expressions are popularly used in theoretical computer science. It is essentially a pattern that matches a set of strings that can be accepted by a Finite State Automaton as its input. (*Regular Languages / Brilliant Math & Science Wiki*, n.d.)

State Diagram

A State Diagram is a diagrammatic representation of a Finite State Machine. While a Finite State Automaton is formally represented using a five-tuple set, the state diagram represents all the information of an FSM using circles and arrows. Each circle denotes a state that the machine exists in, while the arrows represent the transition that takes place from one state to another. Each arrow is labelled with a symbol which denotes the input that is required to make that transition from one state to another. Double lined circles denote the final states that the system could end up in.

RESULTS

Applying Methodology to Sport

Events are the building blocks of the gameplay of any sport. While listing the events that constitute the game play, we came across a question that leads to further categorization of sports:

- Are events of sports independent of each other or not?

The answer to the former question is both yes and no. The above-mentioned events of each sport constitute the game play of that sport. It is on these events that the rules are directly applicable to. While listing out the events that make up the sports, we observe that there are certain sports whose events can be listed down and others where the events of the sports are rather ambiguous and at the mercy of the participants. The latter are those sports where the intention of the sport is not just to merely reach the end state of the game but reach there in the most aesthetic manner possible. Therefore, the sports of the aesthetic kind are composed of events that are at the mercy of the participants, i.e., The participants decide the order and the execution of the events to make the most aesthetic performance while still remaining within the boundaries set by the rules of that sport. In *Art and Sport*, David Best observes this distinction and categorizes sports into two categories - namely Purposive and Aesthetic.

Purposive Sports – “The great majority of sports are what I call "purposive," and in these aesthetic considerations are normally incident.”(Best, 1980) The gameplay of sports of this category serve towards reaching the purpose of the sport, i.e., Winning. It does not matter that the game play is not carried out in the most aesthetic form as long as it is permissible within the rules of the sport.

Aesthetic Sports - “there is a category of sports, which I call "aesthetic," in which the aim cannot intelligibly be specified independently of the manner of achieving it”(Best, 1980) Sports of this kind are those in which the participants aim to execute the gameplay in the most aesthetic manner possible. Examples of this category are Vault, Gymnastics, Synchronized Swimming.

While listing the events of the sport, another question raises - are the events independent of each other? We already know that events are governed by the system of rules of that sport - specifically an event is governed by a few overall game rules as well as the specific event rules

The rules governing an event dictate the outcome of that event, i.e., the rules decide the next state the gameplay is going to enter. While making this observation, we come across a distinction in the way the rules and the events interact in the various games we have considered. In the following sports - Hurdles, Track, Shot Put, Long Jump, Bowling, Badminton, Badminton, Pacman, Weightlifting (Snatch), the rules of the game can dictate the next possible set of events the game play could take. It is still uncertain which exact event the gameplay would unfold in, but the events follow a sequential logic that is set by the rules governing that event. Purposive sports have clearly defined events that serve towards reaching the purpose of the sport, the purpose being winning. These events do not capture the execution of the event; they capture the occurrence of the event which is sufficient for determining the outcome of the game. On the other hand, aesthetic sports like Synchronized swimming, vault etc., do not follow a sequential logic in the events that make up their gameplay.

Aesthetic sports are un-serializable as the events that constitute the gameplay, although are within the confines of the rules, cannot be predicted. The events of aesthetic sports do not require to be ordered sequentially. The outcome of the gameplay is independent of the ordering of the events, unless the ordering of the events contributes to the aesthetic value. In any way the ordering of the events is not strictly governed by the rules of the sport like they do in Purposive sports.

Sports like Synchronized swimming and Diving are judged and scored based on their aesthetic appraisal. The team or participant orders the events of their gameplay/performance in a way that is most aesthetically appealing. From within the confines of the rules of the sport itself, the events can be ordered in a myriad of ways - the only limitation being the team/player's capability to pull it off. Therefore, it is safe to say that Aesthetic sports like Synchronized Swimming and Diving do not follow a sequential logic during

their performance and hence we place them in a category called - ‘Non-Sequential Sports’.

We notice that the distinction Best made - Purposive and Aesthetic also holds true for sports that follow sequential logic and sports that do not. It should be noted here that Best made the categorization based on the objective of the sport rather than on the basis of the presence or absence of sequential logic within the gameplay of these sports. But this distinction holds true if we were to segregate on the basis of sequential logic between the events of the sports as well.

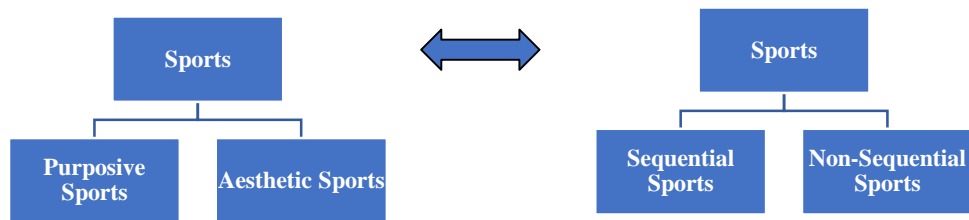


Figure 3: Sports classified as Purposive/Sequential & Aesthetic/Non-Sequential Sports

For sports, whose events and the ordering of events are ambiguous, our endeavor to analyze how the winner emerges stops here. We can observe from the rules and events of the gameplay, that the winner is solely based on the judgement of the aesthetic of the performance. The judgement of the performance is carried out by a panel of judges. Henceforth, we continue our analysis of only those sports where the events can be listed comprehensively and more importantly follow a sequential logic.

Now that we have all the events of the gameplay and the rules that govern each event listed out, we return to the original focus of the paper, i.e., to analyze how a winner is decided in different kinds of sports. Up until now, we have presented the various sports taken into consideration as a list of events and respective rules that govern those events. For further analysis, we represent Sequential sports formally in the form of Finite State Automatons or Finite State Machines.

Sequential Sports as Finite State Machines

In an earlier section, we have made the differentiation between sequential sports and non-sequential sports. We have established that sequential sports have a finite number of events. We have made a comprehensive list of the events that make up the gameplay of the various sports considered. In doing so, we have listed the various states in which the game exists in. We have also listed out the rules of the sports that are applicable to the events that make up the gameplay. By doing so, we have also established the sequence the events could follow to reach one of the final states of the game. Since Sequential Sports display the following aspects:

- Finite number of states the game exists in
- Rules that enable sequential transition function

We can definitively say that Sequential sports can be represented in the form of a Finite State Automaton. Given below are representations of (purposive sports) as Finite State Automata, their respective State Diagrams and Regular Languages.

1. Hurdles

Finite State Automaton (H):

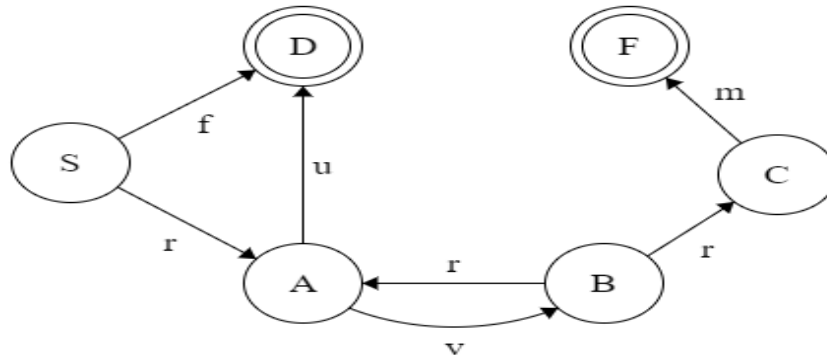
$H = (Q, \Sigma, \delta, q, F)$ where :

- $Q = \{S, A, B, C, D, F\}$
- Q is set of all states that the sport of Hurdles can exist in.
- A - Player is in Running state
- B - Player crosses a hurdle
- C - Player reached the end line successfully while adhering to the rules.
- D - Disqualified
- F - Final State / Score update for Player
- $\Sigma = \{r, f, v, u\}$
- Σ is the set of all actions that the players of Hurdles can possibly take
- r - running
- f - false start
- v - valid jump
- u - faulty jump

- m - measure time taken
- $q = \{S\}$
- q denotes the starting state of Hurdles
- $Q = \{D, F\}$
- Q is the set of all accepting or final states of the sport
- D denotes that the player is disqualified
- F denotes that the player's score is updated.
- δ is the transition function defined by the rules of Hurdles. This function dictates how the state of sport changes from one to another depending upon the action taken by the player.

Table 1: Transition function of Hurdles (δ)

Initial State	Input	Final State
S	f	D
S	r	A
A	u	D
A	v	B
B	r	C
B	r	A
C	m	F

**Figure 4:** State Diagram of Hurdles**Regular Language of Hurdles:**

$$R = f \mid rvr(vr)^*m \mid r(vr)^*u$$

2. Track

Finite State Automaton(T):

$T = (Q, \Sigma, \delta, q, F)$ where :

- $Q = \{S, A, B, D, F\}$ where
 - Q is the set of all possible states that the sport of Track could exist in
 - S - Starting State of the sport
 - A - The players are Running
 - D - Disqualified
 - B - Reached end line while adhering to all the rules
 - F - Player's score updated
- $\Sigma = \{r, f, o\}$ where
 - Σ is the set of all actions that the players of Track could possibly take
 - r - running
 - f - false start
 - o - obstructive movement / moving out of track
 - m - Measurement of Time Taken for Player to reach end line
- $q = \{S\}$ where
 - q denotes the starting state of Track
- $F = \{D, F\}$ where
 - F is the set of all final states of Track
 - D - disqualified
 - F - Player's score is updated.
- δ is the transition function defined by the rules of Track. The actions of the runners define the state changes.

Table 2: Transition function of Track (δ)

Initial State	Input	Final State
S	f	D
S	r	A
A	o	D
A	r	B
A	r	A
B	m	F

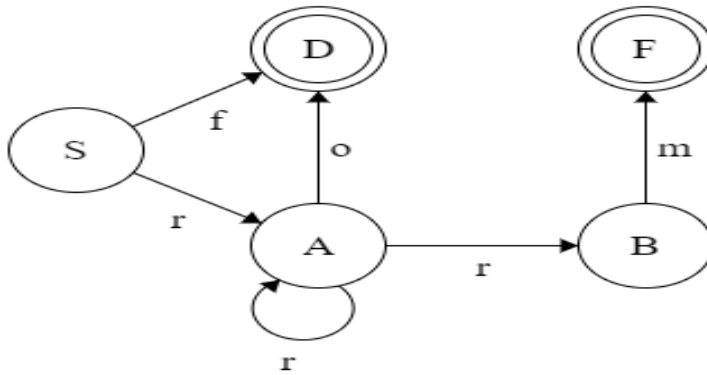


Figure 5: State Diagram of Track

Regular Language of Track:

$$R = f \mid rr^*o \mid rr^*rm$$

3. Shot Put

Finite State Automaton(S):

$S = (Q, \Sigma, \delta, q, F)$ where

- $Q = \{S, A, B, D, F\}$ where
 - Q is the set of all possible states that a game of Shot Put could exist in.
 - S is the starting state where he player takes position
 - A - Shot is thrown
 - B - Shot has landed
 - D - Player is disqualified
 - F - Final State / Player score is updated
- $\Sigma = \{t, f, s, h\}$ where
 - Σ is the set of all actions that the players of Shot Put can take during the gameplay.
 - t - Player throws the shot
 - f - faulty throw by a player
 - s - shot lands
 - h - shot lands outside the sector
 - m - Space is measured between Player and Shot
- $q = \{S\}$ where q is the starting state of the gameplay of Shot Put

- $Q = \{D, F\}$ where
- Q is the set of all final states of the gameplay of Shot Put
- D - Player is disqualified
- δ is the transition function dictated by the rules of Shot Put. The transition function lays out the state changes in the game play of Shot put depending upon the action taken by the player.

Table 3: Transition function of Shotput (δ)

Initial State	Input	Final State
S	f	D
S	t	A
A	h	D
A	s	B
B	m	F

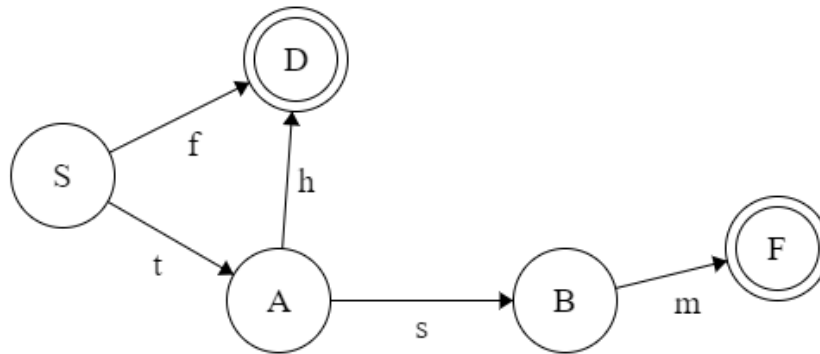


Figure 6: State Diagram of Shot Put

Regular Expression of Shot Put:

$$R = f \mid th \mid tsm$$

4. Long Jump

Finite State Automaton(L):

$L = (Q, \Sigma, \delta, q, F)$ where

- $Q = \{S, A, B, C, D, F\}$ where
- Q is the set of all the possible states the gameplay of Long Jump could exist in.

- S - Starting State, the athlete takes position
- A - Player is running
- B - Player jumps
- C - Player lands
- D – Disqualified
- F - Final State / Player's score updated
- $\Sigma = \{r, j, f, l, h, m\}$ where
- Σ is the set of all actions that the player can take during the course of the gameplay of Long Jump
- r - Player runs
- j - Player jumps
- f - Faulty jump by player
- l - Player lands
- h - Faulty land by the Player
- m - Measure space between foul line and Player landing
- $q = \{S\}$ where q is the starting state of the gameplay of Long Jump
- $Q = \{D, F\}$ where
- Q is the set of all the final states that the gameplay of Long Jump could end up in.
- D - Player is disqualified
- F - Player's score is updated
- δ is the transition function of Long Jump which dictates how the state changes in the game play of Long Jump must take place.

Table 4: Transition function of Long Jump (δ)

Initial State	Input	Final State
S	r	A
A	f	D
A	j	B
B	h	D
B	l	C
C	m	F

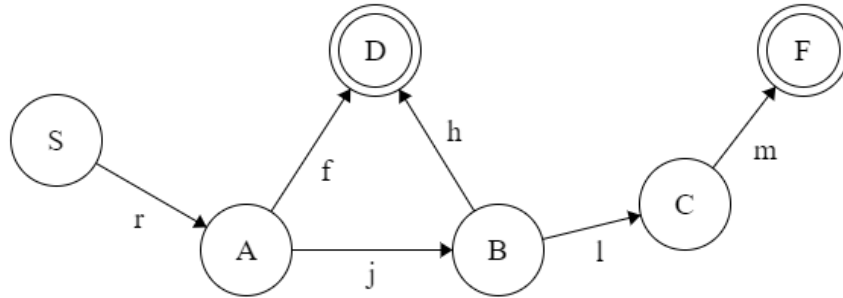


Figure 7: State Diagram Long Jump

Regular Language of Long Jump:

$$R = rf \mid rjh \mid rjlm$$

5. Bowling

Finite State Automaton(B):

$B = (Q, \Sigma, \delta, q, F)$ where

- $Q = \{S, A, B, C, D, E, F, A1, C1, D1, E1, F1\}$ where
 - Q is the set of all states that the game play of Bowling could possibly exist in.
 - S - Starting State. Player takes position.
 - A - Ball is rolling
 - B - Disqualified
 - C - None of the pins left standing on first strike
 - D - Some of the pins left standing on first strike
 - E - All the pins left standing on first strike
 - F - Final Score update
 - $A1$ - Ball is rolling for a second time
 - $C1$ - None of the pins left standing on second strike
 - $D1$ - Some of the pins left standing on second strike
 - $E1$ - All pins left standing after second strike
 - $F1$ - First Score Update
- $\Sigma = \{r, f, e, p1, p2, s\}$ where

- Σ is the set of all actions that are allowed during the gameplay of Bowling
- r - bowler rolls the ball
- f - faulty roll
- e - doesn't hit the pins
- p1 - hits the pins, with few left standing
- P2 - hits the pins , none left standing
- s - score update
- $q = \{S\}$ where q is the starting state of the game play of Bowling
- $Q = \{B, F\}$ where Q is the final states of the gameplay of Bowling
- δ is the transition function that dictates the way the gameplay of Bowling could change states.

Table 5: Transition function of Bowling

Initial State	Input	Final State
S	r	A
S	f	B
A	e	E
A	p1	D
A	p2	C
E	s	F1
D	s	F1
C	s	F
F1	f	B
F1	r	A1
A1	e	E1
A1	p2	C1
A1	p1	D1
E1	s	F
C1	s	F
D1	s	F

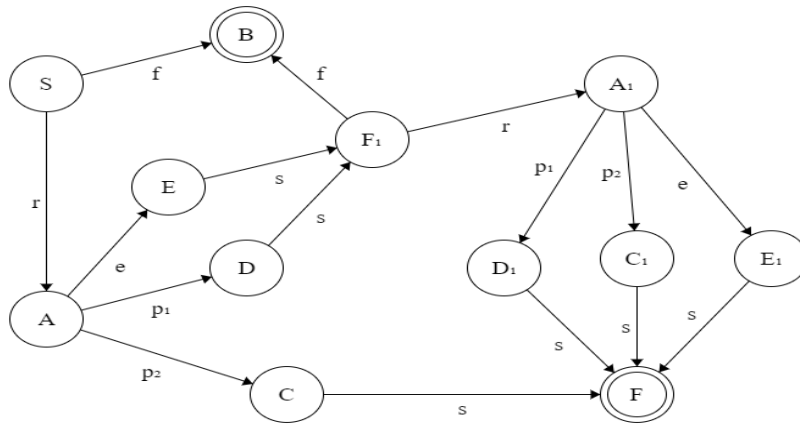


Figure 8: State Diagram of Bowling

Regular Language of Bowling:

R = f | resf | resres | resrp2s | resrp1s | rp1sres | rp1srp2s | rp1srp1s | rp2s | rp1sf

6. Badminton

Finite State Automaton(V):

V = (Q, Σ, δ, q, F) where

- Q = {S, A, B, C, D, E, F, G, H} where
- Q is the set of all the possible states that a game of Badminton could exist in.
- S – Starting State, Player 1 takes position to Serve
- A – Shuttle in rally either due to successful Serve by Player 1 or successful hit with racket by Player 1.
- B – Faulty Serve by Player 1
- C – Shuttle in Rally due to successful hit by Player 2
- D – Rally interrupted due to faulty hit by Player 2
- E – Rally interrupted due to Player 2 letting the Shuttle drop to the ground
- F – Score Update

- G – Rally interrupted due to faulty hit by Player 1
- H – Rally interrupted due to Player 1 letting the Shuttle drop to the ground
- $\Sigma = \{s, f, h1, h2, r1, r2, d1, d2, p1, p2, l\}$ where
- Σ the set of all possible actions by the players of a game of badminton.
- s – Serve
- f - Faulty serve by player 1
- h1 - Shuttle hit by Player 1
- h2 - Shuttle hit by Player 2
- r1 - Faulty hit by Player 1
- r2 - Faulty hit by Player 2
- d1 - Shuttle dropped by Player 1
- d2 - Shuttle dropped by Player 2
- p1 - Point to Player 1
- p2 - Point to Player 2
- l – let
- $q = \{S\}$
- $F = \{F\}$
- δ - Transition Function that dictates how the states of the game changes depending upon the actions taken by the player

Table 6: Transition function of Badminton

Initial State	Input	Final State
S	l	S
S	s	A
S	f	B
A	h2	C
A	d2	E
A	r2	D
B	p2	F
C	r1	G
C	d1	H
E	p1	F
D	p1	F
G	p2	F
H	p2	F

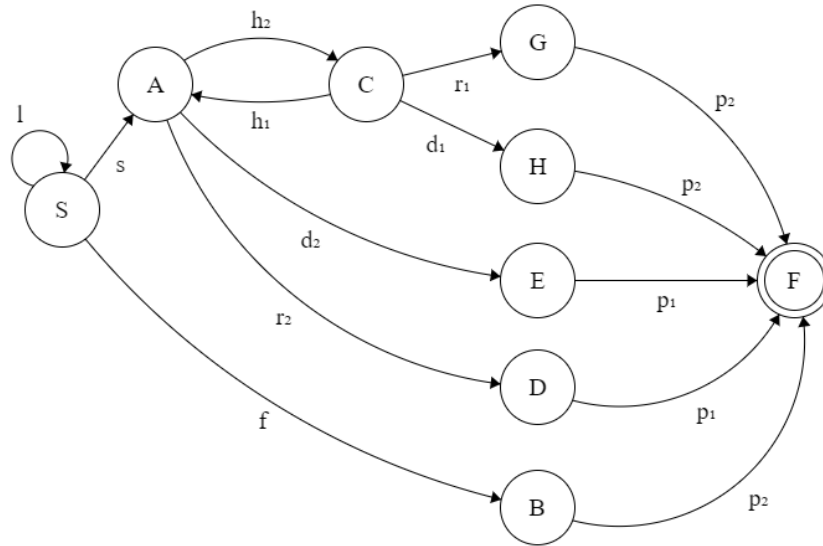


Figure 9: State Diagram of Badminton

Regular Language of Badminton:

lfp2 | ls(h2h1)*d2p1 | ls(h2h1)*r2p1 | ls(h2h1)*h2r1p2 |
 ls(h2h1)*h2d1p2

7. Pacman

Finite State Automaton(P):

P=(Q,Σ,δ,q,F) where

- Q = {S,A,B,C,D,E,F,U} where
- Q is the set of all possible states that a level of Pacman game could exist in.
- S - Starting State, Pacman is in start position
- A - Pac moved its location
- B - Pac consumed the pacdots
- C - Pac stuck at a deadend in the maze
- D - Pac killed by the monster
- E - Pac loses the game (loses all lives)
- F - Pac makes its way out of the maze after eating all the dots

- U - Game score is updated.
- $\Sigma = \{m,o,c,e,k,t,s\}$ where
- Σ is the set of all actions that could take place during the course of the game.
- m - pac is moving
- o - pac hits an obstruction and cannot move
- c - pac changes direction and moves
- e - pac eats dots while moving
- k - pac is killed by monster
- t - pac loses all three lives
- s – Score updated
- $q = \{S\}$
- $F = \{F,E\}$ where
- F is the set of all the final states that the game could end in.
- F - Pac eats all the dots and makes it way out of the maze
- E - Pac loses all lives and loses the game.
- δ - Transition Function that dictates how the states of the game changes depending upon the actions taken by the player.

Table 7: Transition function of Pacman (δ)

Initial State	Input	Final State
S	m	A
S	e	B
S	o	C
S	k	D
A	m	F
A	m	A
A	e	B
A	o	C
A	k	D
B	s	U
C	c	A
C	k	D
D	s	U
U	m	A
U	e	B
U	t	E

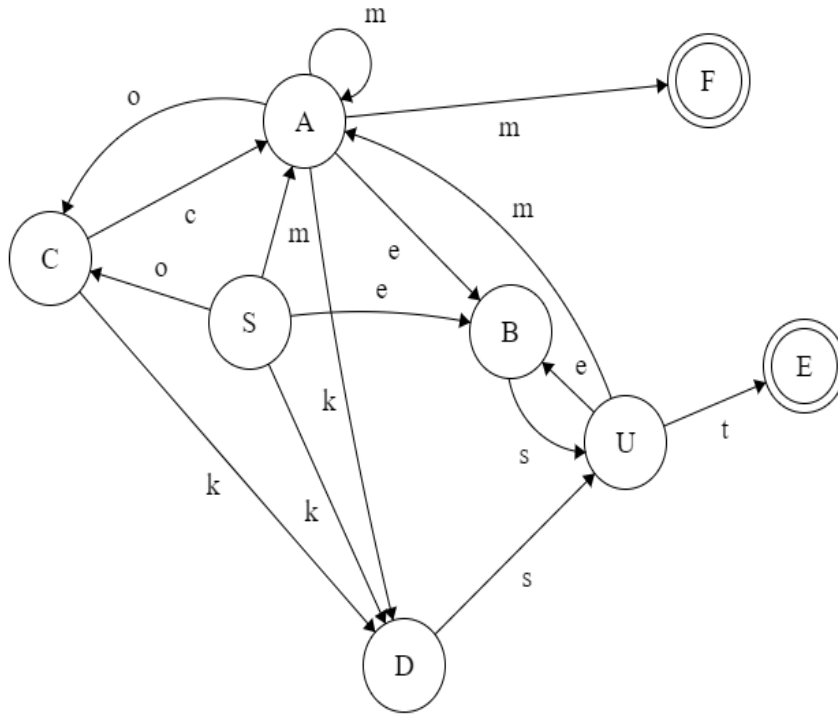


Figure 10: State Diagram of Pacman

Regular Language of Pacman

$$R = m^*(es)^*m^*okst \mid m^*(es)^*m^*(oc)^*m^*kst \mid m^*(es)^*m^*kst \mid m^*(es)^*m^* \mid esm^*(es)^*m^*(oc)^*m^* \mid es(es)^*m^*okst \mid es(es)^*m^*(oc)^*m^*kst$$

8. Weightlifting

Finite State Automaton (W):

$W = (Q, \Sigma, \delta, q, F)$ where

- $Q = \{S, A, B, D, F\}$ where
- Q is the set of all possible states that round of Weightlifting could possibly be in.
- S - Starting State / Lifter takes position
- A - Barbell is lifted

- B - Barbell is lowered
- D - Disqualified
- F - Score is Updated
- $\Sigma = \{l, f, p, q, s\}$ where
- Σ is the set of all actions that a player can taken during the course of the sport.
- l - Player lifts the barbell according to the rules
- f - Player does a faulty lift (does not adhere to the rules while lifting)
- p - Player lowers the barbell according to the rules
- q - Player breaks rules while lowering the barbell
- s - Judges score the player
- $Q = \{S\}$ where q is the starting state of a round of Weightlifting
- $Q = \{D, F\}$ where
- Q is the set of all possible final states of a round of weightlifting
- D - Player is disqualified
- F - Score updated
- δ is the transition function of Weightlifting. It dictates the state changes a round of Weightlifting must take depending upon the actions of the lifter.

Table 8: Transition function of Weightlifting (δ)

Initial State	Input	Final State
S	l	A
S	f	D
A	p	B
A	q	D
B	s	F

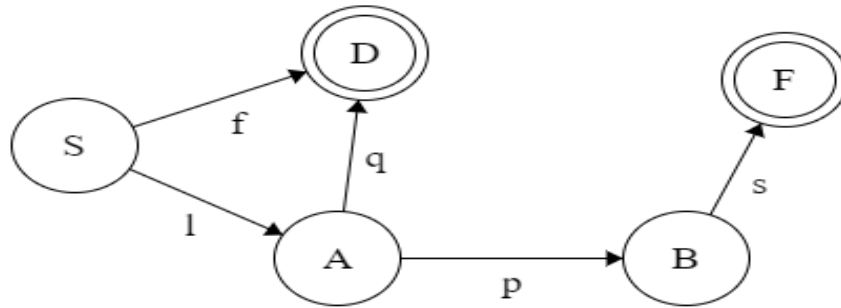


Figure 11: State Diagram of Weightlifting (Snatch)
Regular Language of Weightlifting:
 $R = f \mid lq \mid lps$

Observation based on Formal Presentation

In the previous section, with the help of tools from computation theory, we have represented Sequential sports as Finite State Machines. We have also generated the Regular languages accepted by the Finite State Machines of respective sports. The regular language of a sport is the set of all sequences of events accepted by the sport. In this set, there are some sequences of events that, if followed, would lead to the final state of foul, disqualify, etc. Since this entire exercise is undertaken to analyze how the system of rules pronounces one participant/team as winner, we do not need to consider the regular expressions of a sport that lead to final states like foul and disqualify. We are only interested in the regular expressions of a sport that will help us decide how a winner emerges in that sport.

Therefore, for further analysis, we do not consider the Regular Language(R) of the sports, but we consider a subset of it.

1. Hurdles

Let us assume the Regular Language which expresses all the possible sequences of actions in a game of Hurdles is H(R).

$$H(R) = R = f \mid rvr(vr)^*m \mid rvr(vr)^*u$$

In Hurdles, the expression which leads to the final state where a winner can be announced is represented H(r).

$$H(r) = rvr(vr)^*m$$

2. Track

Let us assume the Regular Language which expresses all the possible sequences of actions in a game of Track is $T(R)$.

$$T(R) = f \mid ro \mid rrm$$

In Track, the expression which leads to the final state where a winner can be announced is represented $T(r)$.

$$T(r) = rrm$$

3. Shot Put

Let us assume the Regular Language which expresses all the possible sequences of actions in a game of Shot Put is $S(R)$.

$$S(R) = f \mid th \mid tsm$$

In Shot Put, the expression which leads to the final state where a winner can be announced is represented $S(r)$.

$$S(r) = tsm$$

4. Long Jump

Let us assume the Regular Language which expresses all the possible sequences of actions in a game of Long Jump is $L(R)$.

$$L(R) = rf \mid rjh \mid rjlm$$

In Long Jump, the expression which leads to the final state where a winner can be announced is represented $L(r)$.

$$L(r) = rjlm$$

5. Bowling

Let us assume the Regular Language which expresses all the possible sequences of actions in a game of Long Jump is $B(R)$.

$$B(R) = f \mid resf \mid resres \mid resrp2s \mid resrp1s \mid rp1sres \mid rp1srp2s \mid rp1srp1s \mid rp2s \mid rp1sf$$

In Bowling, the expression which leads to the final state where a winner can be announced is represented $B(r)$.

$$B(r) = resf \mid resres \mid resrp2s \mid resrp1s \mid rp1sres \mid rp1srp2s \mid rp1srp1s \mid rp2s \mid rp1sf$$

6. Badminton

Let us assume the Regular Language which expresses all the possible sequences of actions in a game of Badminton is $V(R)$.

$$V(R) = lfp2 \mid ls(h2h1)*d2p1 \mid ls(h2h1)*r2p1 \mid ls(h2h1)*h2r1p2 \mid ls(h2h1)*h2d1p2$$

In Badminton, the expression which leads to the final state where a winner can be announced is represented $V(r)$.

$$V(r) = lfp2 \mid ls(h2h1)*d2p1 \mid ls(h2h1)*r2p1 \mid ls(h2h1)*h2r1p2 \mid ls(h2h1)*h2d1p2$$

7. Pacman

Let us assume the Regular Language which expresses all the possible sequences of actions in a game of Pacman is $P(R)$.

$$P(R) = m^*(es)^*m^*okst \mid m^*(es)^*m^*(oc)^*m^*kst \mid m^*(es)^*m^*kst \mid m^*(es)^*m^*esm^*(es)^*m^*(oc)^*m^* \mid es(es)^*m^*okst \mid es(es)^*m^*(oc)^*m^*kst$$

In Pacman, the expression which leads to the final state where a winner can be announced is represented $P(r)$.

$$P(r) = m^*(es)^*m^*okst \mid m^*(es)^*m^*(oc)^*m^*kst \mid m^*(es)^*m^*kst \mid m^*(es)^*m^*esm^*(es)^*m^*(oc)^*m^* \mid es(es)^*m^*okst \mid es(es)^*m^*(oc)^*m^*kst$$

8. Weightlifting

Let us assume the Regular Language which expresses all the possible sequences of actions in a game of weightlifting is $W(R)$.

$$W(R) = f \mid lq \mid lps$$

In Weightlifting, the expression which leads to the final state where a winner can be announced is represented $W(r)$.

$$W(r) = lps$$

Classification based on System of Rules

In this section, we propose a classification of sports, based on the observations that can be drawn from the regular expression of sports that lead to a winning state.

1. Measure Time to decide the winner

The regular expressions of sports that fall under this category have a common final symbol that denotes - measurement of time. It indicates that the winner of these sports are determined based on the time taken by the participants/teams to complete the sequence of events that leads to the final state. In the sports that we have considered, Hurdles and Track are the sports that fall under this category.

2. Measure Space

In this category of sports, the regular expressions of the sports display a common factor in how the winner is determined - the measurement of space. The final character in the regular expressions denotes measurement of space to determine the winner. The sports that fall under this category are Shot put and Long Jump. In both these sports, measurement of space plays a pivotal role in determining the winner.

3. Compare Points

In this category of sports, the assessment of the players or teams is done via the points acquired by the player/team. The last character of the strings in the regular expressions of these sports are similar and it denotes comparison of points between teams or participants. The points can be acquired in the form of a score, foul, let, goal etc. The scoring system of these sports is governed by the rules of that particular sport. At the end of the gameplay of the sport, the points accumulated by the participants or teams are compared. The rules of the games dictate how this comparison is to be made - some sports announce the team with the highest points as the winner while other sports go the other way.

This category of sports contains a wide variety of sports and the way in which the points are accumulated varies - therefore it has been further classified into sub-categories. The classification of the sub-classes has been done with the help of the graphical representation of sports.

a. Win Condition

Let us consider the regular expressions of Bowling and analyze at what instances the score of a team or a participant is updated. For this analysis, we consider $B(r)$ - the expressions that lead to a score update.

$$B(r) = \text{resf} \mid \text{resres} \mid \text{resrp2s} \mid \text{resrp1s} \mid \text{rp1sres} \mid \text{rp1srp2s} \mid \text{rp1srp1s} \mid \text{rp2s} \mid \text{rp1sf}$$

's' which indicates Score Update is always preceded by actions of the same team/player even though there can be two or more teams/players involved in the gameplay of Bowling. In the above given regular expression, we can notice that the score update s is preceded by e(ball doesn't hit the

pins) or p2(ball hits the pins, with few left standing) or p1(ball hits the pin, with none left standing) which are all actions taken by the same team/player.

An observation we can draw from the regular expressions is that the score update of a team or player is independent of the other team/player's performance. The score is updated solely based upon the performance of one's own performance. Another inference we can draw is that the score of either team is not updated at a final state which leads to foul. Only by performing the goal of the game, the score is updated. Therefore, we termed this category - Update score on Win Condition and Bowling is a sport that is central to this category.

b. Lose Condition

Let us consider the regular expressions of Badminton and analyze at what points the scores are updated for either of the teams. For this analysis, we consider $V(r)$ - the expressions that lead to a score update.

$$V(r) = lfp2 \mid ls(h2h1)*d2p1 \mid ls(h2h1)*r2p1 \mid ls(h2h1)*h2r1p2 \mid ls(h2h1)*h2d1p2$$

'p2' which point to Team 2 is always preceded by an action by Team 1. In the regular expression given above we can notice that 'p2' is preceded by 'f' (faulty serve by Team 1) or 'r1' (Faulty hit by Team 1) or 'd1' (Team 1 let the shuttle drop)

Unlike the previous category, the update of score of one team is entirely dependent on the performance of the opponent's team. It is when the opponent's team commits a faulty serve, drops the shuttle to the ground, faulty hit by players(f,d,r), the score is updated.

The goal of the sport, to keep returning the shuttle to the opponent while adhering to the rules does not lead to any score update. Only when an opponent's team slips up from following the rules, the score is updated. Hence, this category is labelled as the Score Update on Lose Condition.

c. Win & Lose Condition

Let us consider the regular expressions of the game Pacman, to analyze at what points the score of the game is updated. For this analysis, we consider $P(r)$ – the regular expressions of Pacman that lead to a score update.

$$P(r) = m^*(es)*m^*okst \mid m^*(es)*m^*(oc)*m^*kst \mid m^*(es)*m^*kst \mid m^*(es)*m^* \mid esm^*(es)*m^*(oc)*m^* \mid es(es)*m^*okst \mid es(es)*m^*(oc)*m^*kst$$

In this game, there are two kinds of instances where the score is updated - one where the monster kills Pacman(k), the other when Pacman eats Pac dots(e). Both the actions of 'e' and 'k' lead to a Score update. The action 'k-killed by Monster' falls under Lose condition whereas the action 'e-Pac eats Pac dots' falls under Win Condition. Therefore, the score update of Pacman is dependent both on the performance of a player as well as that of the opponents. The win condition of the score update is indicated by Pacman eating something (point increase) whereas the lose condition is indicated by the monster killing Pacman (loses a life). Therefore, this category is called Score Update on Win and Lose Condition.

4. Panel of Judges

In this class of sports, the assessment of the players is done by a panel of judges. The players or teams should adhere to the rules governing the sport to be eligible for an assessment by a panel of judges. The score allotted by the panel of judges is decided upon by taking into consideration various factors like posture, aesthetic appeal, synchronization etc. The scoring procedure followed in this category is similar to that of aesthetic sports, but this category was included in Purposive sports as the events of the sports that follow under this category follow a sequential logic. For instance, take Weightlifting.

$$W(r) = lps$$

The events that precede s (Score update) are l (lifts the barbell) and p (lowers the barbell). These events do not have much flexibility of variation in the way they are performed. They are also governed by Event Rules like lifter must not pause during the lifting of the barbell, lifter must not stop the upward motion while the pull, the barbell must not make contact with the lifter's head or shoulders while the upward motion of the pull etc., which are mentioned in detail in an earlier section. Since these events follow a sequential logic, they are categorized under Sequential sports. The form and the ease with which the actions 'l' (lifting the barbell) and 'p' (lowering the barbell) are assessed by the judges and scores are announced accordingly.

CONCLUSION

We have started our analysis of sport with the classification proposed by Caillois in *Man, Play and Games* where sports fall under a category of games called Agon. Agon refers to those games where competition to establish a player/team as the superior is central to the game. Caillois points out that Sport is only one manifestation of Agon - a cultural form of Agon to be precise. Through the course of this paper, we have analyzed the structures of eight different Agon games to understand how these structures produce a winner. The classification we propose at the end of this analysis is therefore based on the structure of sports.

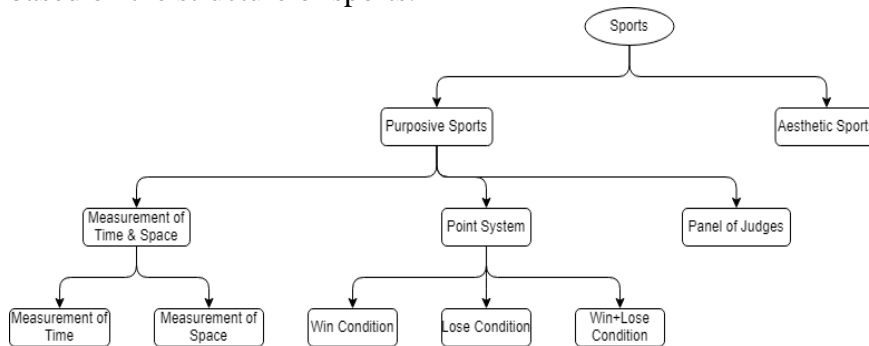


Figure 12: Proposed Classification of Sports

Although the structures of the games of different categories may be different, they all convey the idea of competition and winning through their structures. They have in common the idea of Agon. Now this leads us to the inquiry if there are other structures of competition that are not sports but fit into this categorization. While sports are an institutionalized form of competition that have come to become a cultural phenomenon, we also engage in competition outside of this sphere. At a personal level, we engage in competition with our siblings, peers and colleagues. These kinds of games are intimate and do not follow a rigid structure to arrive at a winner. We employ different tactics in different scenarios that they could be placed in all of the categories at different times. It is when these personal games lend themselves to refinement, in the process becoming institutionalized versions that are accepted in social life, that a rigid structure of competition comes

into place and thereby susceptible to classification. Some of the structures of competitions that have gone through this process of institutionalization are competitive examinations. Although competitive exams are far from being classified as sports, we cannot deny the resemblance they have with the structure of competitive games. Depending upon the marking scheme of an examination, they could easily fit into one of the categories mentioned above. Then there are structures of economic competition like auction sales, the stock exchange, different market structures (monopolies, oligopolies etc.) that function on the idea of Agon. In the future, we intend to explore those structures that are not strictly sport but lend themselves the quality of Agon.

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